ECONOMETRIC MODELING OF THE SURVEY RESEARCH CENTER PRICE EXPECTATIONS DATA

By

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TABLE OF CONTENTS

PAGE

ACKNOWLEDGMENTS ii

ABSTRACT iv

CHAPTER I A SURVEY OF PREVIOUS RESEARCH ON THE SRC PRICE EXPECTATIONS DATA. 1

1.1 Introduction. 1
1.2 Questions and Responses 4
1.3 Previous Estimates. 6
1.4 Outline 16

CHAPTER II A THEORETICAL MODEL OF THE BENEFITS OF PRICE LEVEL PREDICTORS 19

2.1 The Basic Model 19
2.2 Extensions. 25
2.3 Summary 28

CHAPTER III ESTIMATION OF EXPECTED INFLATION MODELS. 29

3.1 Alternatives. 29
3.2 Observability 34
3.3 Estimation With Rounding. 35
3.4 Estimation With Costless Predictors 42
3.5 Summary 51

CHAPTER IV ESTIMATION OF A MACROECONOMIC MODEL USING THE SRC EXPECTED INFLATION SERIES. 53

4.1 The Macroeconomic Model 53
4.2 Estimation of the Macroeconomic Model . . . 56
4.3 Summary and Conclusions 60

BIBLIOGRAPHY 62

BIOGRAPHICAL SKETCH. 64

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This study considers econometric modeling of the Survey Research Center (SRC) price expectations data. There are two major difficulties in modeling the SRC data. First, there are two types of censoring. All predictions of deflation and some predictions of inflation are censored. The two-stage estimation methods used previously with these data are inconsistent because the partial censoring of the inflation predictions has been ignored, leading to inconsistent first-stage estimates. Maximum-likelihood estimation methods that account for both types of censoring are presented. Second, the data appear to have been generated by a discontinuous distribution. The difference limen, a minimum perceivable inflation rate, has been used in previous work to construct such a districation. However, the problems in perception are restricted to the neighborhood of zero inflation, and the resulting estimates of the difference limen are inconsistent with observed predictions.

Two alternatives to the difference limen are considered. The first alternative is based upon the possibility that different responses have been rounded to different degrees. This model is statistically similar to a difference limen model where only some of the people sampled are imperceptive. The advantage of the rounding model is that it can explain discontinuities that occur at points other than just zero inflation, and because of the rounding interpretations, eliminates the need to estimate the range of imperceptibility.

The estimates of this model also provide a means of examining the performance of the cross-section variance of expected inflation as a measure of subjective uncertainty. The estimates suggest that in the absence of a direct measure of uncertainty, the cross-section variance may be a useful alternative.

The second alternative considers the impact of costless predictors on the observed distribution of responses. This model provides a measure of expected inflation for a relatively informed group. While the expectations of this informed group are a better predictor of the actual inflation rate, estimation of a macroeconomic model suggests that they are no more relevant for modeling purposes than the expectations of the general population.

CHAPTER I
A SURVEY OF PREVIOUS RESEARCH ON THE SURVEY RESEARCH CENTER PRICE EXPECTATIONS DATA

1.1 Introduction

Expected inflation has emerged as an important variable in theoretical models of the economy. A variety of methods have been used to obtain measures of expected inflation for empirical testing of these models. These measures can be described as either direct or indirect measures of expected inflation. Indirect measures require specification of the relationship between expected inflation and some set of observable variables. Adaptive expectations and rational expectations are examples of methods that rely upon indirect measures. The disadvantage of an indirect measure is that any test conducted is really a joint test of the model and the assumed method of expectation formation. Direct measures are obtained from survey data. Because survey data yield observable expectations, assumptions regarding the method of expectation formation are unnecessary. Thus, direct measures are a potentially useful alternative to indirect measures.

There are two data sets which serve as the basis for most direct measures of expected inflation. The first is the data set collected by Joseph A. Livingston from a panel of economists. The second is the data set collected by the Survey Research Center (SRC) of the University of Michigan.

The SRC data have several advantages relative to the Livingston data. First, unlike the Livingston data, the SRC data are a random sample of the general population. This is an advantage, because the sample is drawn from the population whose behavior is being modeled. This limitation of the Livingston data has been recognized previously.

> The drawback of the Livingston data is that they represent the expectations of professional forecasters (observers), not the expectations of consumers or businessmen (actors). (Juster and Comment, 1978, p. 1)

There is some evidence that suggests this problem may have serious consequences. Estimates of the variance of expected inflation from the Livingston data are substantially smaller than the corresponding estimates from the SRC data. This observation led to the following conclusion by Maddala, Fishe, and Lahiri (1981).

> It is important to use economy-wide estimates... when analyzing the relationship between the variance of the expected rate of inflation and the variance of the actual rate of inflation...Otherwise, by using only experts' opinions one is likely to understate the true level of uncertainty in the economy. (Maddala, Fishe, and Lahiri, 1981, p. 9)

There is also some question about the forecast horizon of the Livingston data. This is because Livingston asked for predictions of the Consumer Price Index, and the date of the most-recent price index at the time of the forecast is unclear. Because the Consumer Price Index is reported with a lag, the end-of-period forecast of the price index is being made with knowledge of the most-recent price index, rather than with knowledge of the current price index. This extends the forecast horizon back to the date of the most-recent index. Carlson (1977) argues that the actual forecast horizon is two months longer than indicated by Livingston. This problem is avoided with the SRC data, because the inflation rate is predicted. Because the individual is not asked to forecast the Consumer Price Index with a specified base period, he can choose the current period to be the base period, avoiding the problem of not knowing the current price index for some other base period.

Finally, the SRC data set is composed of large samples. The average sample size is approximately one thousand a month. The large samples provide accurate estimates, and are important when using these data, because only asymptotic properties are known for the estimation methods to be used.

In summary, the estimates obtained from the SRC data are likely to be useful, because they require no assumptions about how expectations are formed, they represent the expectations of the general population, and they are based on large samples. Still, there are some difficulties associated with use of the SRC data. First, the observations are doubly censored. Point predictions were not obtained from those expecting deflation. And point predictions of inflation were obtained only from a subset of those expecting inflation. Thus, despite the direct nature of the observations of expected inflation, assumptions about the cross-sectional distribution of expected inflation are still necessary in order to obtain estimates. Second, the observations do not appear to have been generated by a continuous distribution. Certain observations occur with much greater frequency, relative to neighboring observations, than would be expected to occur with a continuous distribution. Some method of explaining this discontinuity is required. Finally, it has been argued that the presence of uninformed individuals gives rise to unexploited opportunities for profit in, for example, securities markets. A few informed speculators with large resources would be able to arbitrage away these profits, and in doing so would make market interest rates reflect their own informed expectations rather than the expectations of the general population. If this were the case, a survey of a few informed individuals, such as the Livingston survey, might be preferred to a more broadly based survey.

Two explanations for the discontinuity in distribution will be considered. The first explanation will provide a means of examining the performance of the cross-section variance of expected inflation as a proxy for the uncertainty with which individual expectations are held. The second explanation will result in a measure of expected inflation for a relatively informed group. And while it is not clear that this relatively informed group is the relevant group for modeling purposes, neither is it clear that the economists sampled by Livingston are the relevant group. These explanations are addressed following a discussion of the nature of the SRC questions regarding expected inflation, and a survey of previous research with the SRC data.

1.2 Questions and Responses

The data to be used in this dissertation were gathered on a monthly basis from January 1978 to February 1980. Each month, respondents were asked two questions.

> During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now? (Juster and Comment, 1978, p. 3)

An individual could either not respond, or respond "up," "down," "stay where they are," or "do not know" to this question. Those who responded up were asked a second question.

> By what percent do you expect prices to go up, on average, during the next 12 months? (Juster and Comment, 1978, p. 3)

The individual could either not respond, respond with a prediction, or respond "do not know" to this second question.

The responses to these questions can be grouped into five categories. A description of the categories follows.

No Response. This category includes those who did not respond to the first question, and those who answered "do not know" in response to the first question. Because these responses yield no information about expected inflation, they will be ignored. Fortunately, only about one percent of the sample fell into this category.

Deflation. This category contains those who answered "down" in response to the first question. All that is known about this set of observations is that expected inflation is negative.

No Inflation. This category contains those who answered "prices stay where they are" in response to the first question.

Inflation. This category contains those who answered "up" in response to the first question, and either did not respond to the second question, or responded "do not know". All that is known about this set of observations is that expected inflation is positive.

Percent Inflation. This category contains those who answered "up" in response to the first question, and gave a prediction of the inflation rate in response to the second question.

This categorization illustrates the first problem encountered when using the SRC data. The data are doubly censored: only the sign of expected inflation is known for the Deflation and Inflation categories.

In addition to the data set just described, there is an earlier data set collected by the SRC. This data set will be important in the survey of previous research with the SRC data. Prior to 1978 the SRC data was collected on a quarterly basis. The questions themselves have followed a lengthy evolution, and are documented in Juster and Comment (1978). However, the questions and possible responses are very similar to those just described for the current data set.

From 1948 through the first quarter of 1966, only the first question was asked. For this time period, only qualitative information about expected inflation is known. All predictions of inflation or deflation are censored. The categorization of responses given earlier still holds, with the additional restriction that there are no responses in the Percent Inflation category.

From the second quarter of 1966 through 1977, both questions were asked. However, point predictions given in response to the second question were recorded only as falling into certain subcategories. The responses for this time period can be categorized exactly as before, except the Percent Inflation category now contains the five subcategories, 1-2 percent, 3-4 percent, 5 percent, 6-9 percent, and 10+ percent. Just as with the latter data set, all predictions of deflation and some of the predictions of inflation are censored.

Table 1.1 contains the sample frequencies of each category in the current data set. This table shows the importance of accounting for the censored data. On average, 15 percent of the data was censored; 13 percent of this censoring occurred in the Inflation category. This particular category ranged from a low of 8 percent to a high of 24 percent. While the Inflation category represents the most frequent form of censoring in the 1978-1980 data, as well as in the earlier data, it has been ignored in all previous uses of the SRC data.

1.3 Previous Estimates

The estimates obtained from the earlier data by Juster (1973), deMenil and Bhalla (1975), and Juster and Comment (1978), do not adequately account for the censored data. Using the post-1966 data, deMenil and Bhalla (1975) calculate sample means and variances using the

Table 1.1
Percent of 1978-1980 Data in Each Category

	Deflation	No Inflation	Inflation	Percent Inflation
78:1	2.2	9.9	18.5	69.4
78:2	1.0	9.7	15.1	74.2
78:3	0.6	9.8	18.4	71.2
78:4	1.0	5.5	21.9	71.6
78:5	1.7	10.4	10.4	77.5
78:6	1.6	7.3	16.7	74.4
78:7	2.3	8.2	15.5	74.0
78:8	1.7	9.7	9.5	79.1
78:9	1.5	10.2	17.1	71.2
78:10	1.1	8.7	17.2	73.0
78:11	1.0	9.4	15.1	74.5
78:12	1.2	4.9	23.8	70.1
79:1	1.3	5.0	15.3	78.4
79:2	1.8	7.9	8.5	81.8
79:3	0.7	3.9	14.6	80.8
79:4	0.9	5.3	14.8	79.0
79:5	1.1	6.3	8.3	84.3
79:6	1.4	8.3	9.7	80.6
79:7	2.0	8.7	8.3	81.0
79:8	2.5	10.4	9.6	77.5
79:9	2.6	9.3	11.5	76.6
79:10	2.5	10.2	14.0	73.3
79:11	2.5	10.2	9.9	77.4
79:12	2.0	11.6	6.8	79.6
80:1	1.6	7.6	9.0	81.8
80:2	0.7	7.5	8.4	83.4
Average	1.6	8.3	13.4	76.7

midpoints of the categorical responses as inflation forecasts, with zero used for those who expect deflation, or no inflation. The same procedure is used by Juster (1973), and Juster and Comment (1978), except -3 percent is used as the mean expectation of deflation.

> This basically implies that deMenil and Bhalla are only using the responses from the "up" category to estimate μ [mean expected inflation]. By doing so they introduce a bias in their estimates which is proportional to σ [the cross-section standard deviation of expected inflation]. (Maddala, Fishe, and Lahiri, 1981, p. 14)

deMenil and Bhälla (1975) use the biased estimates of μ and σ obtained from the post-1966 data to estimate the parameters of the following equations.

$$\frac{\mu}{\sigma} = \beta \text{ BAL} \tag{1.1}$$

$$\sigma = \alpha_0 + \alpha_1 \mu + \alpha_2 \text{MOOD} \tag{1.2}$$

where BAL is the percentage of the sample expecting inflation minus the percentage expecting deflation, and MOOD is an index of consumer sentiment available from the SRC data. The values of BAL and MOOD from the pre-1966 data set, and the estimated equations are used to obtain estimates of μ and σ back to 1948.

> Juster and Comment (1978) and deMenil and Bahalla (1975) work backward from these estimates and the responses to the qualitative question to generate forecasts back to 1948. The linking mechanism is somewhat arbitrary, but essentially involves a scheme whereby the difference between the percentage expecting prices to increase and the percentage expecting prices to decrease is related to these weighted averages [the sample means and variances]. The main problem with this procedure is that it is arbitrary, with no statistical basis. (Maddala, Fishe, and Lahiri, 1981, p. 13)

Working with the earlier data Fishe and Lahiri (1981), and Maddala, Fishe, and Lahiri (1981) provide methods that account for the censoring that occurs in the Deflation category. Recognizing that the number of individuals responding that they expected no inflation was disproportionately large relative to the neighboring categories, the "difference limen" was introduced to explain this discontinuity at zero expected inflation. The difference limen is a minimum perceivable rate of inflation or deflation. That is, when the absolute rate of expected inflation is sufficiently small, individuals cannot perceive any difference from zero inflation and will respond "no inflation". The No Inflation category is then interpreted as an interval of inflation rates centered upon zero inflation.

Let Π^* denote expected inflation, and let δ denote the different limen. Let $F(\cdot)$ be the continuous distribution of Π^* without problems in perception, and let $f(\cdot)$ be the corresponding density. With the difference limen, the distribution of expected inflation becomes

$$P(\Pi^* \leq c) = \begin{cases} F(c) & c < -\delta \\ F(-\delta) & -\delta \leq c < 0 \\ F(\delta) & 0 \leq c \leq \delta \\ F(c) & c > \delta \end{cases} \tag{1.3}$$

The difference limen has transformed the continuous distribution of responses into one that is discontinuous at zero. Therefore,

$$P(\Pi^* = c) = \begin{cases} f(c) & |c| > \delta \\ F(\delta) - F(-\delta) & c = 0 \\ 0 & |c| \leq \delta \text{ and } c \neq 0 \end{cases} \tag{1.4}$$

There should be no observations other than zero within the difference limen interval.

Given the preceding distributional assumptions, Fishe and Lahiri (1981) and Maddala, Fishe, and Lahiri (1981) make the following assumptions regarding the observability of Π^*. Observe,

$$\begin{array}{ll} \Pi^* & \text{if } \Pi^* > \delta \\ \text{"no inflation"} & \text{if } -\delta \leq \Pi^* \leq \delta\text{, and} \\ \text{"deflation"} & \text{if } \Pi^* < -\delta. \end{array} \qquad (1.5)$$

From (1.3) and (1.4) the probabilities of these responses are

$$\begin{array}{ll} P(\Pi^*) & = f(\Pi^*) \\ P(\text{"no inflation"}) & = F(\delta) - F(-\delta) \\ P(\text{"deflation"}) & = F(-\delta) \end{array} \qquad (1.6)$$

Let S_1 be the set of N_1 elements in the Deflation category, S_2 be the set of N_2 elements in the No Inflation category, S_3 be the set of N_3 elements in the Inflation category, and S_4 be the set of N_4 elements in the Percent Inflation category. Then the likelihood function for these responses is

$$L = \Pi_{S1} F(-\delta)\ \Pi_{S2}[F(\delta) - F(-\delta)]\ \Pi_{S4} f(\Pi^*) \qquad (1.7)$$

Two things should be noted about this likelihood function. First, observations in the Inflation category (those who responded "up" but did not provide a point prediction) are being ignored. Second, the midpoints of the categorical responses in the Percent Inflation category are being used as point predictions.

The estimation methods of Fishe and Lahiri (1981) and Maddala, Fishe, and Lahiri (1981) are very similar. Both assume that cross-section variation in expected inflation is the result of a disturbance term. That is,

$$\Pi_i^* = \mu + e_i \qquad (1.8)$$

where Π_i^* is expected inflation for each individual, μ is mean expected inflation across individuals, and e_i is the individual specific disturbance term. Both obtain the likelihood function (1.7), and both use two-stage methods to obtain estimates. The major difference between these papers liesin the choice of distribution for e_i.

Maddala, Fishe, and Lahiri (1981) assume that e_i is normally distributed with mean zero and variance σ^2. Let

$$B = \frac{-\delta-\mu}{\sigma} \text{ and } D = \frac{\delta-\mu}{\sigma} \tag{1.9}$$

The likelihood function corresponding to (1.7) is then

$$L = \Pi_{S1}\Phi(B)\ \Pi_{S2}[\Phi(D) - \Phi(B)]\ \Pi_{S4}\frac{1}{\sigma}\ \phi\left(\frac{\Pi^*-\mu}{\sigma}\right) \tag{1.10}$$

where $\Phi(\cdot)$ is the standard normal distribution, and $\phi(\cdot)$ is the standard normal density. Equations (1.9) may be solved for

$$\frac{\mu}{\sigma} = \frac{-(B+D)}{2} \text{ and } \frac{\delta}{\sigma} = \frac{(D-B)}{2} \tag{1.11}$$

In the first stage only the directional information is used. Sample proportions are used as estimates of $\Phi(D)$ and $\Phi(B)$. That is,

$$\hat{\Phi}(B) = \frac{N_1}{N} \text{ and } \hat{\Phi}(D) = \frac{N_1+N_2}{N} \tag{1.12}$$

These can be used to obtain estimates of B and D.

$$\hat{B} = \Phi^{-1}\left(\frac{N_1}{N}\right) \text{ and } \hat{D} = \phi^{-1}\left(\frac{N_1+N_2}{N}\right) \tag{1.13}$$

In the second stage, the quantitative responses in the Percent Inflation category are incorporated into the estimation process through the following equation.

$$\begin{aligned} E(\Pi^*|\Pi^* > \delta) &= \mu + E(e_i|e_i > \delta - \mu) \\ &= \mu + \sigma\frac{\phi(D)}{1-\Phi(D)} \end{aligned} \tag{1.14}$$

Substituting for σ from (1.11) and solving gives

$$\mu = E(\Pi^*|\Pi^* > \delta)\ [1 - \frac{2}{(B+D)}\frac{\phi(D)}{1-\Phi(D)}]^{-1} \tag{1.15}$$

The sample mean of the predictions in the Percent Inflation category, $\overline{\Pi}^*$, can be used to estimate $E(\Pi^*|\Pi^* > \delta)$, and the estimates from the first stage given in (1.13) can be used to calculate the adjustment factor. That is,

$$\hat{\mu} = \overline{\Pi}^*\ [1 - \frac{2}{(\hat{B}+\hat{D})}\frac{\phi(\hat{D})}{1-\phi(\hat{D})}]^{-1} \tag{1.16}$$

Equations (1.11) may now be solved for estimates of σ and δ.

Fishe and Lahiri (1981) assume instead that e_i is distributed sech^2. Let

$$J = -\delta - \mu \text{ and } K = \delta - \mu \tag{1.17}$$

The likelihood function corresponding to (1.7) is then

$$L = \Pi_{S1}G(J)\ \Pi_{S2}[G(K) - G(J)]\ \Pi_{S4}g(\Pi^*-\mu) \tag{1.18}$$

where $G(\cdot)$ is the logistic function, and $g(\cdot)$ is the corresponding density. Equations (1.17) may be solved for

$$\mu = \frac{-(J+K)}{2} \text{ and } \delta = \frac{K-J}{2} \tag{1.19}$$

Once again, the sample proportions are used in the first stage to obtain estimates of $G(J)$ and $G(K)$.

$$\hat{G}(J) = \frac{N_1}{N} = P_1 \text{ and } \hat{G}(K) = \frac{N_1+N_2}{N} = P_2 \tag{1.20}$$

These equations are solved to give

$$\hat{J} = G^{-1}\left(\frac{N_1}{N}\right) = \log\left(\frac{P_1}{1-P_1}\right) \text{ and}$$

$$\hat{K} = G^{-1}\left(\frac{N_1+N_2}{N}\right) = \log\left(\frac{P_2}{1-P_2}\right) \qquad (1.21)$$

Substituting these estimates into (1.19) gives the first-stage estimates.

In the second stage, the quantitative responses are incorporated through the following equation.

$$E(\Pi^*|\Pi^* > \delta) = \mu + E(e_i|e_i > \delta - \mu)$$

$$= \mu - \frac{[KG(K) + \log(1-G(K))]}{1-G(K)} \qquad (1.22)$$

The sample mean of the quantitative responses is used to estimate $E(\Pi^*|\Pi^* > \delta)$, and the first-stage estimates are used to calculate $E(e_i|e_i > \delta - \mu)$. This gives

$$\hat{\mu} = \bar{\Pi}^* + \frac{[\hat{K}G(\hat{K}) + \log(1-G(\hat{K}))]}{1-G(\hat{K})} \qquad (1.23)$$

In order to simplify exposition and emphasize the similarity of the estimation methods in each paper, the assumptions of Maddala, Fishe, and Lahiri (1981) have been retained in the discussion of the estimation methods of Fishe and Lahiri (1981). The actual differences in the assumptions, and their impact on the estimation process are as follows. First, Fishe and Lahiri (1981) transform the inflation rate forecasts into price level forecasts, denoted P*. That is,

$$P^* = P(1 + \Pi^*) \qquad (1.24)$$

where P is determined by the choice of base period of the price index. Applying the transformation in (1.24) to the basic equation for Π^* given in (1.8), while retaining the remaining assumptions, will result in mean price level estimates equivalent to the inflation rate estimates transformed by (1.24). Second, the mean price level each period is assumed to depend on aggregate but not individual-specific explanatory variables. That is,

$$\mu_t = Z_t\gamma \tag{1.25}$$

Substituting (1.25) into (1.17) and (1.22) we obtain

$$\hat{J}_t = -\delta - Z_t\gamma \text{ and } \hat{K}_t = \delta - Z_t\gamma \tag{1.26}$$

and

$$\bar{\Pi}^*_t + \frac{[\hat{K}_t G(\hat{K}_t) + \log(1-G(\hat{K}_t))]}{1 - G(\hat{K}_t)} = Z_t\gamma \tag{1.27}$$

Instead of solving equations (1.17) and using (1.23) to obtain estimates of μ and δ for each period separately, least-squares methods are applied to (1.26) and (1.27) to obtain estimates of γ and δ.

Consistency of the two-stage methods just presented requires the use of consistent estimates in the first stage. Because Fishe and Lahiri (1981) and Maddala, Fishe, and Lahiri (1981) ignore the responses in the Inflation category (those who responded "up" but did not give a point prediction), their first-stage estimates are not consistent. When the responses in the Inflation category are ignored, the sample proportions will underestimate the probability of the "up" responses and overestimate the probability of the "down" and "same" responses. This results in downward bias in the estimates of μ. Because there are more responses in the Inflation category than in the Deflation and No Inflation categories combined, it is likely to be of practical importance that these responses be accounted for.

Also, in my opinion, the difference limen is an unsatisfactory explanation of the large number of responses in the No Inflation category.

First, individuals are assumed to have problems in perception only in the neighborhood of zero inflation. In other words, the individual cannot see the difference between no inflation and inflation of 0.1 percent. Yet he can distinguish perfectly between 10 percent inflation, and inflation of 10.1 percent.

Second, no observations other than zero should occur within the difference limen interval. However, the estimates of Maddala, Fishe, and Lahiri (1981) are such that this occurs for almost every period of the data set. As suggested in Maddala, Fishe,and Lahiri (1981), this inconsistency can be overcome by assuming the sample is composed of perceptive and imperceptive individuals. Only the imperceptive individuals are subject to the difference limen. If λ is the probability of randomly selecting a perceptive individual, the distribution of responses after the imposition of the difference limen is

$$P(\Pi^* \leq c) = \begin{cases} F(c) & c < -\delta \\ \lambda F(c) + (1-\lambda)F(-\delta) & -\delta \leq c < 0 \\ \lambda F(c) + (1-\lambda)F(\delta) & 0 \leq c \leq \delta \\ F(c) & c > \delta \end{cases} \qquad (1.28)$$

The resulting distribution is still discontinuous at zero, but as (1.29) indicates, there is no longer a restriction on non-zero observations falling in the difference limen interval.

$$P(\Pi^* = c) = \begin{cases} f(c) & |c| > \delta \\ (1-\lambda)[F(\delta) - F(-\delta)] & c = 0 \\ \lambda f(c) & |c| \leq \delta \text{ and } c \neq 0 \end{cases} \qquad (1.29)$$

Finally, and more fundamentally, it is not clear that expectations are subject to the same problems of perception as observable variables. Expectations are subjective statistics, conceived by the individuals, yet not perceived by them. While it seems reasonable that an individual faced with an observable set of prices might have problems perceiving small changes in the price level, the existence of a difference limen for observable prices does not imply the same holds for expectations of unobservable future prices. Perhaps a general example may be helpful in illustrating the difference. While an individual may not be able to tell the difference between a temperature of 77 degrees and one of 78, this does not mean that when the same individual is asked to predict the temperature, he does not know the difference between a prediction of 77 degrees and one of 78.

1.4 Outline

To summarize the focus of the thesis, there are two important problems to be addressed when using the SRC data. First, there are two types of censoring. All predictions of deflation and some predictions of inflation are censored. The two-stage estimation methods used previously with this data set are inconsistent because the partial censoring of the inflation predictions has been ignored, leading to inconsistent first-stage estimates. Second, the data appear to have been generated by a discontinuous distribution. The difference limen has been used in previous work to construct such a distribution. However, the problems in perception are restricted to the neighborhood of zero, and the resulting estimates of the difference limen are inconsistent with observed predictions. Two alternatives to the difference limen will be considered. The first alternative is based upon the possibility that different responses have been rounded to different degrees. The estimates of this model will provide a means of examining the performance of the cross-section variance of expected inflation as a measure of the uncertainty with which individual expectations are held. The second alternative considers the impact of costless predictors on the observed distribution of responses. This model will provide a measure of expected inflation for a relatively informed group, composed of those who are not using the costless predictors considered.

Chapter II uses a two-period model to examine the benefits of price level prediction. Incentives to gain price level information are provided by the existence of bond holdings, and by the assumption that individuals must precommit on their labor supply decisions. This model will show that the benefits of any predictor approach zero as the elasticity of current expenditure approaches zero. Given the almost costless publication of economic statistics by the media, the individual may use the most-recent price level as a costless predictor of the current price level, or he may use the most-recent inflation rate as a costless predictor of the current inflation rate. These predictors provide an essentially costless base with which to compare costly predictors. One would suspect that some individuals will be sufficiently inelastic in current expenditure, that given the costless alternative, the benefits of costly predictors will not justify the cost of acquisition. Others will be sufficiently elastic in current expenditure that the benefits due to the improved performance of a costly predictor will justify the cost of acquisition. Thus, we have the possibility of some individuals' using the simple costless predictors discussed earlier.

Chapter III considers how the distribution of responses is influenced by rounding of responses, and by the use of costless predictors. Maximum-likelihood estimation methods that account for both types of censoring are presented.

In Chapter IV, the expected inflation series is used to estimate a macroeconomic model of the economy. Both the initial and long-run impacts of expected inflation on nominal interest rates can be estimated. The existence of an expectation adjusted Phillips curve can also be tested with this model.

CHAPTER II
A THEORETICAL MODEL OF THE BENEFITS OF PRICE LEVEL PREDICTORS

2.1 The Basic Model

In order to consider the benefits of a price level predictor, two important decisions requiring future price level information will be incorporated into a theoretical model of consumer choice. These are the use of nominally denominated bonds to transfer purchasing power over time and the purchase or sale of a good at a stated nominal price in a future period.

A two-period model with these decisions may be constructed by assuming that individuals can buy or sell a one period bond that is a promise to pay one dollar in the second period, and by assuming that individuals must precommit in their labor supply decisions for the second period. The individual gets utility from current and future goods consumption, and from current and future leisure. At the beginning of the first period, the individual is assumed to know the current price level, the current wage, the current interest rate, the second period wage, and the mean future price level. There is uncertainty about the future price level. Assume the individual is an expected utility maximizer who must choose current consumption, current labor, and future labor. Given his choice of current consumption and current labor, the budget constraint determines bond holding. Given bond holding and the choice of future labor, the expected future price level determines expected future consumption, while the actual future price level determines actual future consumption. The problem is

$$\max E[V(X_1, X_2, L_1, L_2)] \text{ subject to}$$

$$P_1X_1 + W_1L_1 + \frac{B}{(1+r)} = W_1\bar{L} \text{ and}$$

$$P_2X_2 + W_2L_2 = W_2\bar{L} + B \qquad (2.1)$$

where V is the utility function, X_1 is current goods consumption with price level P_1, X_2 is future goods consumption with price level P_2, L_1 is leisure in the current period, L_2 is leisure in the future period, $\bar{L}$ is maximum leisure in either period, W_1 is the nominal wage in the current period, W_2 is the nominal wage in the future period, and B is the number of bonds with price $\frac{1}{(1+r)}$. The two constraints may be written as the single constraint

$$P_1X_1 + \frac{P_2}{(1+r)} X_2 = W_1(\bar{L} - L_1) + \frac{W_2}{(1+r)} (\bar{L} - L_2). \qquad (2.2)$$

That is, the present value of goods consumption equals the present value of income.

Assuming the individual is a price taker, the first-order conditions are

$$EV_1 = E[V_2 \frac{(1+r)P_1}{P_2}]$$

$$EV_3 = E[V_2 \frac{(1+r)W_1}{P_2}]$$

$$EV_4 = E[V_2 \frac{W_2}{P_2}] \qquad (2.3)$$

These conditions and the budget constraint may be solved for demands for each good. The demands will be functions of the relative prices in (2.3). With a perfect predictor of prices, the demands are as above, but with the actual price level substituted for the expected price level.

It is somewhat cumbersome to keep track of X_1, L_1, and L_2 separately. In order to simplify the analysis note that the budget constraint may be written as

$$\left[P_1X_1 + W_1L_1 + \frac{W_2L_2}{(1+r)}\right] + \frac{P_2}{(1+r)}X_2 = \left[W_1\overline{L} + \frac{W_2}{(1+r)}\overline{L}\right] \tag{2.4}$$

Letting the bracketed terms be Z and Y respectively, letting $S = \frac{1}{P_2}$ and solving gives

$$X_2 = (1+r)SY - (1+r)SZ \tag{2.5}$$

Because changes in the future price level do not alter the relative prices of current consumption, current leisure, and future leisure, Z can be considered a composite good. This good is expenditure on items that must be contracted for in the current period, henceforth current expenditure. Since Z is a composite good, a utility function can be obtained from V such that the solution to

$$\max U(X_2,Z) \text{ subject to } X_2 = (1+r)SY - (1+r)SZ \tag{2.6}$$

given W_1, W_2 and r, corresponds to the solution to the previous problem. For proof of this statement, see Diewert (1974).

Consider the benefits of a perfect predictor over knowledge of the mean future price level. Figure 2.1 depicts the situation where the actual price level is less than the expected price level. Using the expected price level, the individual will purchase Z of the composite good and expect to purchase X_2 in future goods. But because of this error in expectation he will be able to purchase X_2'' in future goods. The individual will be better off than he expected to be, but not as well off as if he knew the actual price level. With a perfect predictor, the individual would have chosen point B. Since the individual must

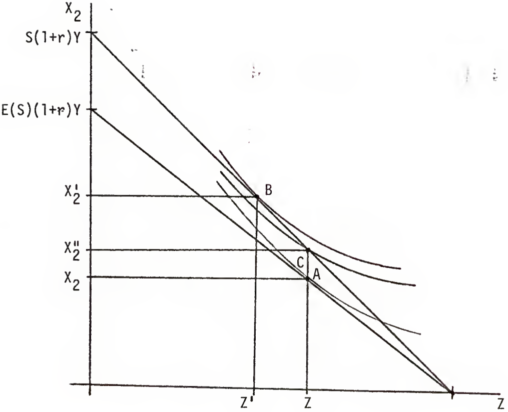

Figure 2.1
Choice With Different Predictors

always be on the actual constraint, the lost utility from the use of the expected price level is measured by comparing point B with the realized point C. Note that the utility loss associated with the use of the expected price level is determined by the change in the composite good Z. If current expenditure had remained unchanged, there would be no advantage to perfect information.

To find the utility loss associated with use of the expected price level, take a second-order Taylor's series expansion around the perfect-information solution, requiring the individual be on the actual constraint.

$$\begin{aligned} dU &= [-(1+r)SU_1 + U_2]dZ \\ &\quad + \tfrac{1}{2}[(1+r)^2S^2U_{11} - 2(1+r)SU_{12} + U_{22}]dZ^2 \\ &= \tfrac{1}{2}[(1+r)^2S^2U_{11} - 2(1+r)SU_{12} + U_{22}]dZ^2 \qquad (2.7) \end{aligned}$$

The second equality follows at the perfect-information solution, because of the first-order condition. This term is negative by the second-order condition. Using the definition of Z from (2.4), dZ is given as

$$dZ = [\partial Z/\partial(1+r)S](1+r)dS = N(1+r)dS \qquad (2.8)$$

The term N is the slope of the current expenditure function. The gain in any realized state from the use of the perfect predictor rather than the mean price level is

$$\begin{aligned} G[S,E(S)] &= -dU \\ &= -\tfrac{1}{2}[(1+r)^2S^2U_{11} - 2(1+r)SU_{12} + U_{22}]N^2(1+r)^2[S-E(S)]^2 \end{aligned}$$

$$(2.9)$$

For simplicity in finding the expected utility gain assume that N, U_{11}, U_{12} and U_{22} are constants.

$$E[G(S,E(S))] = -\tfrac{1}{2}(1+r)^4N^2U_{11}\ E[S^2(S-E(S))^2] + (1+r)^3N^2U_{12}E[S(S-E(S))^2] - \tfrac{1}{2}(1+r)^2N^2U_{22}E[(S-E(S))^2] \tag{2.10}$$

If $S \sim N(\mu,\tau^2)$ then

$$E[G(S,E(S))] = -\tfrac{1}{2}(1+r)^2N^2[U_{11}(1+r)^2(\mu^2+3\tau^2) - 2U_{12}(1+r)\mu + U_{22}]\tau^2 \tag{2.11}$$

See Lindgren (1976) for the necessary expectations. This term is positive when the second-order conditions are satisfied. Also note that $\partial E(G)/\partial\tau^2 > 0$ and $\partial E(G)/\partial N > 0$. Finally, as N approaches zero, the benefits also approach zero. Thus, the greater the variability of S, and the more elastic current expenditure, the greater the benefits of perfect information.

If the perfect information has nominal cost C, the constraint becomes

$$X_2 = (1+r)SY - (1+r)SZ - (1+r)SC \tag{2.12}$$

The utility loss after the imposition of the cost is

$$dU = [-(1+r)SU_1 + U_2](\partial Z/\partial C)dC - (1+r)SU_1dC \tag{2.13}$$

Note that $dC = C$ here. The expected utility loss of given up C dollars is

$$E(dU) = [-(1+r)\mu U_1 + U_2](\partial Z/\partial C)dC - (1+r)\mu U_1 dC$$

$$= -(1+r)\mu U_1 C \qquad (2.14)$$

where the second equality follows from the first-order conditions. This is just the expected utility of the goods given up in order to invest in the perfect predictor.

The individual will invest in the perfect predictor if the expected benefits exceed the expected cost. As N approaches zero, the benefits also approach zero. Those individuals with inelastic current expenditure will not find it advantageous to invest.

The perfect predictor just considered corresponds to the rational expectations case of perfect foresight. The results obtained suggest what one would have suspected intuitively, that in the presence of costs the rational expectation may differ from the expected utility maximizing expectation. Thus, observed expectations should not be expected to have the properties of rational expectations. For tests of rationality see Fackler and Stanhouse (1977), Mullineaux (1978), and Pearce (1979).

2.2 Extensions

The previous section considered the benefits of perfect information. This is rarely, if ever, available. Instead, assume the individual can obtain a conditional expectation, $\hat{S}$, based upon the relevant variables of a jointly normal model. Following the steps in the previous section, the expected utility gain of $\hat{S}$ over E(S) is

$$E[G(\hat{S},E(S))] = E[G(S,E(S))] - E[G(S,\hat{S})]$$

$$= -\frac{1}{2}(1+r)^2N^2[U_{11}(1+r)^2(\mu^2+5\tau^2-2\tau^2R^2) - 2U_{12}(1+r)\mu+U_{22}]\tau^2R^2 \qquad (2.15)$$

where R^2 is defined in the usual way, and measures the ability of $\hat{S}$ to explain variation in S. This result is consistent with those obtained in the previous section. If $\hat{S}$ is a perfect predictor, R^2 is one and (2.15) reduces to (2.11). Also, $\partial E(G)/\partial\tau^2 > 0$, $\partial E(G)/\partial N > 0$, and $\partial E(G)/\partial R^2 > 0$. That is, the greater the variability of S, the greater the ability of $\hat{S}$ to explain variation in S, and the more elastic current expenditure, the greater the benefits of the predictor $\hat{S}$. Once again, as N approaches zero, the expected benefits approach zero.

Up to this point, the individual was assumed to have knowledge of E(S). A more realistic approach would be to make E(S) available at some cost, and to use S_{t-1}, the most-recent information, as the costless benchmark. This is more realistic due to the almost costless publication of statistics by the media. The expected utility gain of $\hat{S}$ over S_{t-1} is

$$\begin{aligned} E[G(\hat{S},S_{t-1})] &= E[G(S,S_{t-1})] - E[G(S,\hat{S})] \\ &= -\tfrac{1}{2}N^2(1+r)^2[U_{11}(1+r)^2(\mu^2+5\tau^2-2\tau^2R^2) - 2U_{12}(1+r)\mu+U_{22}]\tau^2R^2 \\ &\quad -\tfrac{1}{2}N^2(1+r)^2[U_{11}(1+r)^2(\mu+\tau^2) - 2U_{12}(1+r)\mu+U_{22}]B^2 \\ &\quad - 2N^2(1+r)^2[U_{11}(1+r)^2\mu - U_{12}(1+r)]\tau^2B \end{aligned} \tag{2.16}$$

where $B = [E(S) - S_{t-1}]$ is the bias of S_{t-1}. This result is also consistent with the previous expressions. If $S_{t-1} = E(S)$ then $B = 0$ and (2.16) reduces to (2.15). The first two terms are positive. The sign of the third term cannot be determined. The first term is the expected utility gain of the conditional expectation over the mean. The remaining terms involve the bias of S_{t-1}.

In general, the use of S_{t-1} rather than E(S) as the costless benchmark will drive up the value of the costly predictors. But as before, the benefits approach zero as N approaches zero. Thus, the central result remains: those individuals with sufficiently inelastic current expenditure will find the benefits of predictors other than the most recent information do not justify the costs.

Consider how the use of the current price level to predict the future price level will affect wage demands in each period. That is, given the individual has completed his current contract, and is now recontracting for the next two periods, what change in nominal wages will induce him to supply the same amount of labor as under the previous contract? When using the current price level to predict the future price level, the labor supplies obtained from (2.3) reduce to

$$\overline{L} - L_i = \ell^i((1+r), \frac{W_1(1+r)}{P_1}, \frac{W_2}{P_1}) \quad i = 1,2 \tag{2.17}$$

If we were interested in the behavior of aggregates, such as total employment, overlapping generations could be assumed to determine the relationship between changes in expected inflation and changes in total employment. In this application we are interested in the behavior of the individual. So long as the contracts established in the first period are enforced, the individual will recontract every two periods. As the individual recontracts every two periods, there is an updated value for P_1.

$$d\ell^i = \ell_2^i d\,(\frac{W_1(1+r)}{P_1}) + \ell_3^i d\,(\frac{W_2}{P_1}) \quad i = 1,2 \tag{2.18}$$

We wish to determine the change in nominal wages that will induce the individual to supply the same amount of labor as under the previous contract. This will occur if

$$d\left(\frac{W_1(1+r)}{P_1}\right) = 0 \text{ and } d\left(\frac{W_2}{P_1}\right) = 0 \qquad (2.19)$$

But these equations imply

$$\frac{dW_1}{W_1} = \frac{dP_1}{P_1} \text{ and } \frac{dW_2}{W_2} = \frac{dP_1}{P_1} \qquad (2.20)$$

Thus, use of the most-recent price level as a costless predictor results in wage demands each renewal period that just compensate for the most-recent inflation.

2.3 Summary

In this chapter a two-period model was used to examine the benefits of price level prediction. The individual was required to make decisions on current and future values of goods consumption and labor supply. The incentive to gain price level information was provided by assuming the individual had to precommit on his second-period labor supply at a stated nominal wage, and by allowing the existence of a nominally denominated bond with which to transfer purchasing power between periods.

This model provides results to be used in the first section of the next chapter. The benefits of any predictor will approach zero as the elasticity of current expenditure approaches zero. And the use of the most-recent price level as a costless predictor results in wage demands each period that just compensate for the most-recent inflation.

CHAPTER III
ESTIMATION OF EXPECTED INFLATION MODELS

3.1 Alternatives

The first alternative to the difference limen is based upon the possibility that some of the individuals have given rounded responses. The choice of rounding units is arbitrary and is likely to vary across individuals. However, there is a strong convention toward rounding in powers of ten. Despite the explicit categorization of the inflation rate predictions in the earlier SRC data, Fishe and Lahiri (1981), and Maddala, Fishe, and Lahiri (1981), treat responses not subject to the difference limen as point predictions. To facilitate comparison with these papers, the first model to be presented will have this same structure. Observations will be considered as either unrounded or rounded to the nearest ten. This choice has been motivated by the accumulation points in the sample frequencies. The sample frequencies of integer multiples of ten are disproportionately large.

Let the symbol I denote the set of integer multiples of ten, and let [c] denote the largest element of I less than or equal to c. Let λ denote the probability of randomly selecting someone that does not round, and $(1-\lambda)$ denote the probability of randomly selecting someone that rounds. Let $F(\cdot)$ denote the distribution of expected inflation prior to rounding, and let $f(\cdot)$ be the corresponding density. After rounding, the distribution of expected inflation is

$$P(\Pi^* \leq c) = \lambda F(c) + (1-\lambda)F([c] + 5) \tag{3.1}$$

This distribution is discontinuous at each integer multiple of ten.

$$P(\Pi^* = c) = \begin{cases} \lambda f(c) & \text{if } c \notin 1 \\ (1-\lambda)F(c+5) - F(c-5)] & \text{if } c \in 1 \end{cases} \tag{3.2}$$

This model remedies the problems of the difference limen that were outlined in the previous section. First, the rounding is not restricted to the neighborhood of zero inflation. The possibility of rounding is extended to the full range of observation. Second, as with the generalized difference limen model with perceptive and imperceptive individuals, there are no restrictions regarding observability near points of discontinuity. In fact, the rounding model in (3.2) and the generalized difference limen model in (1.29) are quite similar statistically. The advantage of the rounding model is that it extends the discontinuities over the full range of observation, and because of the rounding interpretation, eliminates the need to estimate δ. Finally, while there is the statistical similarity of treating certain responses as interval observations, in my opinion the rounding explanation is more appealing.

The probabilities given in (3.2) indicate that the preceding model treats all observations at the integer multiples of ten as rounded responses. This is because the unrounded responses are observations of a continuous variable. This seems somewhat restrictive, and can be corrected by taking into account the fact that only integer responses were given.

The following model treats the observations as either rounded to the integer, or rounded to the ten. In this case, λ is now the probability of rounding to the integer. The distribution of expected inflation is now

$$P(\Pi^* \leq c) = \lambda F(c + .5) + (1-\lambda)F([c] + 5) \tag{3.3}$$

where c is the integer. This distribution is discontinuous at the integers.

$$P(\Pi^* = c) = \begin{cases} \lambda[F(c + .5) - F(c - .5)] & c \notin I \\ \lambda[F(c + .5) - F(c - .5)] & \\ \quad + (1-\lambda)[F(c + 5) - F(c - 5)] & c \in I \end{cases} \tag{3.4}$$

Because

$$F(c + .5) - F(c - .5) \underset{\sim}{\sim} f(c) \tag{3.5}$$

it seems unlikely that the parameter estimates, other than λ, will differ much between either of the preceding models. Substituting (3.5) into (3.4) reveals that (3.4) differs from (3.2) only by the addition of the term $\lambda f(c)$ at the integer multiples of ten. Because some of the observations at the integer multiples of ten can now be attributed to responses rounded to the integer, rather than just responses rounded to the ten, λ is likely to be larger.

The second alternative to the difference limen is based upon the existence of costless predictors of the price level or inflation rate. The incentive for use of the costless predictors, and their effect on behavior has been discussed in detail in Chapter II. In summary, an individual whose current expenditure is sufficiently inelastic with respect to the price level will not find the benefits of a costly predictor worthwhile and will resort to costless predictors. This is because the benefits of any predictor will approach zero as the elasticity of current expenditure approaches zero. What "sufficiently inelastic"means depends upon the cost of the predictor.

The first costless estimator to be considered is the use of the most-recent price level as a predictor of the current price level. This corresponds to predicting no inflation. This predictor will be rather inaccurate in the face of an ongoing inflation. However, for individuals using a costless predictor there is little benefit to anticipating inflation. Instead, these individuals react to inflation. Because an updated price level estimate is received each period, this predictor leads to increased wage demands each period that just compensate the individual for the most-recent inflation. This wage adjustment process is quite popular in reality.

These considerations change the way in which the data must be handled. When randomly sampling from the population there is some probability of selecting an individual who is using the most-recent price level as a costless predictor, and who will always predict no inflation.

Let q be the probability of randomly selecting an individual using the costless predictor. Let $J_i = 1$ if the individual invests in a costly predictor, and $J_i = 0$ otherwise. The observed predictions from the population are $\Pi_i^* J_i$. The separate components of $\Pi_i^* J_i$ are not observed. There are now two reasons why an individual might respond "no inflation". First, he is using the costless predictor, most-recent price level. Second, he has invested in a costly predictor, and the prediction is for no inflation.

Let $F(\cdot)$ be the continuous distribution of responses for those who invest in a costly predictor, and let $f(\cdot)$ be the corresponding density. The distribution of responses from the population is

$$P(\Pi_i^*J_i \leq c) = \begin{cases} P(J_i=1,\ \Pi_i^* \leq c) = (1-q)F(c) & \text{if } c < 0 \\ P(J_i=1,\ \Pi_i^* \leq c) + P(J_i=0) = (1-q)F(c) + q & \text{if } c \geq 0 \end{cases} \tag{3.6}$$

This distribution is discontinuous at zero. This gives

$$P(\Pi_i^*J_i = c) = \begin{cases} (1-q)f(c) & \text{if } c \neq 0 \\ q & \text{if } c = 0 \end{cases} \tag{3.7}$$

Because the distribution of responses from those who invested in a costly predictor was assumed to be continuous, all predictions of no inflation are identified as resulting from the costless predictor. Once again, the fact that only integer responses were given can be used to correct this. With integer responses the distribution becomes,

$$P(\Pi_i^*J_i \leq c) = \begin{cases} (1-q)F(c + .5) & c < 0 \\ (1-q)F(c + .5) + q & \text{if } c \geq 0 \end{cases} \tag{3.8}$$

with corresponding probabilities

$$P(\Pi_i^*J_i = c) = \begin{cases} (1-q)[F(c + .5)-F(c - .5)] & \text{if } c \neq 0 \\ q + (1-q)[F(c + .5)-F(c - .5)] & \text{if } c = 0 \end{cases} \tag{3.9}$$

where c is an integer.

It is unlikely that the parameter estimates, other than q, will differ much between these models. Once again, (3.5) can be used to see that (3.9) differs from (3.7) only by the addition of the term $(1-q)f(c)$ at $c = 0$. Because some of the responses at zero can now be attributed to the costly predictors, rather than just the costless predictor, q is likely to be smaller.

Another costless estimator is the use of the most-recent inflation rate to predict the current inflation rate. This corresponds to predicting last period's inflation rate to reoccur. Let p be the probability of randomly selecting an individual using the most-recent inflation rate as a costless predictor. For $\Pi_{t-1} > 0$, the distribution of responses from the population becomes

$$P(\Pi_i^* \leq c) = \begin{cases} (1-q-p)F(c) & \text{if } c < 0 \\ (1-q-p)F(c) + q & \text{if } 0 \leq c < \Pi_{t-1} \\ (1-q-p)F(c) + q + p & \text{if } c \geq \Pi_{t-1} \end{cases} \tag{3.10}$$

This distribution is discontinuous at zero and Π_{t-1}. This gives

$$P(\Pi^* = c) = \begin{cases} (1-q-p)f(c) & \text{if } c \neq 0 \text{ and } c \neq \Pi_{t-1} \\ q & \text{if } c = 0 \\ p & \text{if } c = \Pi_{t-1} \end{cases} \tag{3.11}$$

This distribution can also be adjusted to account for the integer responses. Using the approximation (3.5) again, the resulting probabilities will differ from (3.11) only by the addition of the term $(1-q-p)f(c)$ at $c = 0$ and $c = \Pi_{t-1}$.

3.2 Observability

The estimation methods to be presented will account for the censoring that occurs in the Deflation and Inflation categories. The censoring that occurs in the Deflation category is imposed by the structure of the questions. There is no allowance for point predictions of deflation. The censoring that occurs in the Inflation category is determined by the individual. An opportunity to give a point prediction was provided. The simplist way to handle this censoring is to assume there is some probability, denoted b, of randomly selecting an individual that is willing to give a point prediction.

With this assumption, observe

$$\begin{array}{lll} \Pi^* & \text{with probability } b & \text{if } \Pi^* > 0 \\ \text{"inflation"} & \text{with probability } (1-b) & \text{if } \Pi^* > 0 \\ \text{"no inflation"} & & \text{if } \Pi^* = 0 \\ \text{"deflation"} & & \text{if } \Pi^* < 0 \end{array} \qquad (3.12)$$

The distribution of expectations for the alternatives described in the first section of this chapter will be used to obtain the probabilities of the responses in (3.12). The resulting probabilities will be used to apply maximum-likelihood methods of estimation. The use of maximum-likelihood methods requires the distribution of the underlying predictions be specified. From now on, $F(\cdot)$ will be the normal distribution with mean μ and variance σ^2, and $f(\cdot)$ the corresponding density.

3.3 Estimation With Rounding

The first alternative considers the possibility of rounded responses. For the model with rounded and unrounded responses, the probabilities in (3.1) and (3.2) give

$$\begin{array}{ll} P(\Pi^*) = \begin{cases} b\lambda f(\Pi^*) & \text{if } \Pi^* \notin I \\ b(1-\lambda)[F(\Pi^*+5) - F(\Pi^*-5)] & \text{if } \Pi^* \in I \end{cases} \\ P(\text{"inflation"}) \quad (1-b)[\lambda[1-F(0)] + (1-\lambda)[1-F(5)]] \\ P(\text{"no inflation"}) = (1-\lambda)[F(5) - F(-5)] \\ P(\text{"deflation"}) = \lambda F(0) + (1-\lambda)F(-5) \end{array} \qquad (3.13)$$

Let S4A be the set of elements in S4 that are not elements of I. Let S4B be the set of elements of S4 that are elements of I. The log likelihood for this model is

$$\begin{aligned}\log L = {} & \Sigma_{S1} \log[\lambda F(0) + (1-\lambda)F(-5)] \\ & + \Sigma_{S2} \log(1-\lambda) + \log[F(5) - F(-5)] \\ & + \Sigma_{S3} \log(1-b) + \log[\lambda[1-F(0)] + (1-\lambda)[1-F(5)]] \\ & + \Sigma_{S4A} \log b + \log\lambda + \log f(\Pi^*) \\ & + \Sigma_{S4B} \log b + \log(1-\lambda) + \log[F(\Pi^*+5) - F(\Pi^*-5)]\end{aligned} \tag{3.14}$$

The first derivatives are

$$\partial \log L/\partial b = \Sigma_{S3} \frac{-1}{(1-b)} + \Sigma_{S4A} \frac{1}{b} + \Sigma_{S4B} \frac{1}{b} \tag{3.15}$$

$$\begin{aligned}\partial \log L/\partial \lambda = {} & \Sigma_{S1} \frac{F(0)-F(-5)}{\lambda F(0)+(1-\lambda)F(-5)} \\ & + \Sigma_{S2} \frac{-1}{(1-\lambda)} \\ & + \Sigma_{S3} \frac{[1-F(0)]-[1-F(5)]}{\lambda[1-F(0)]+(1-\lambda)[1-F(5)]} \\ & + \Sigma_{S4A} \frac{1}{\lambda} \\ & + \Sigma_{S4B} \frac{-1}{(1-\lambda)}\end{aligned} \tag{3.16}$$

$$\begin{aligned}\partial \log L/\partial \mu = {} & \Sigma_{S1} \frac{-\lambda f(0)-(1-\lambda)f(-5)}{\lambda F(0)+(1-\lambda)F(-5)} \\ & + \Sigma_{S2} \frac{-f(5)+f(-5)}{F(5)-F(-5)} \\ & + \Sigma_{S3} \frac{\lambda f(0)+(1-\lambda)f(5)}{\lambda[1-F(0)]+(1-\lambda)[1-F(5)]} \\ & + \Sigma_{S4A} \frac{\Pi^*-\mu}{\sigma^2} \\ & + \Sigma_{S4B} \frac{-f(\Pi^*+5)+f(\Pi^*-5)}{F(\Pi^*+5)-F(\Pi^*-5)}\end{aligned} \tag{3.17}$$

$$
\begin{aligned}
\partial \log L/\partial\sigma^2 = {} & \Sigma_{S1}\,\frac{-\frac{1}{2}\lambda(\frac{0-\mu}{\sigma^2})f(0) - \frac{1}{2}(1-\lambda)(\frac{-5-\mu}{\sigma^2})f(-5)}{\lambda F(0) + (1-\lambda)F(-5)} \\
& + \Sigma_{S2}\,\frac{-\frac{1}{2}(\frac{5-\mu}{\sigma^2})f(5) + \frac{1}{2}(\frac{-5-\mu}{\sigma^2})f(-5)}{F(5) - F(-5)} \\
& + \Sigma_{S3}\,\frac{\frac{1}{2}\lambda(\frac{0-\mu}{\sigma^2})f(0) + \frac{1}{2}(1-\lambda)(\frac{5-\mu}{\sigma^2})f(5)}{\lambda[1-F(0)] + (1-\lambda)[1-F(5)]} \\
& + \Sigma_{S4A}\,\frac{-1}{2\sigma^2} + \frac{(\Pi^*-\mu)^2}{2\sigma^4} \\
& + \Sigma_{S4B}\,\frac{-\frac{1}{2}(\frac{\Pi^*+5-\mu}{\sigma^2})f(\Pi^*+5) + \frac{1}{2}(\frac{\Pi^*-5-\mu}{\sigma^2})f(\Pi^*-5)}{F(\Pi^*+5) - F(\Pi^*-5)}
\end{aligned}
\qquad (3.18)
$$

Equation (3.15) can be solved for b as the sample proportion of those responding "up" that gave a point prediction. The remaining equations were solved using the nonlinear optimization procedure of Berndt, Hall, Hall, and Hausman (1974). The estimates obtained are presented in Table 3.1. These estimates are of the expected sign and magnitudes. Notice, however, that the expected inflation series consistently underpredicts the actual inflation rate. This was also true of the earlier data set, as indicated in Fishe and Lahiri (1981).

In order to check the sensitivity of the estimates to the assumption that the observations were either rounded to the ten or unrounded, estimates will be obtained for the integer response model. Substituting the probabilities of the integer response model in (3.3) and (3.4) into (3.12) gives the probabilities of the responses as

$$P(\Pi^*) = \begin{cases} b\lambda[F(\Pi^*+.5) - F(\Pi^*-.5)] & \text{if } \Pi^* \notin I \\ b[\lambda[F(\Pi^*+.5) - F(\Pi^*-.5)] & \\ \quad + (1-\lambda)[F(\Pi^*+5) - F(\Pi^*-5)]] & \text{if } \Pi^* \in I \end{cases}$$

$$\begin{aligned} P(\text{"inflation"}) &= (1-b)[\lambda[1-F(.5)] + (1-\lambda)[1-F(5)]] \\ P(\text{"no inflation"}) &= \lambda[F(.5) - F(-.5)] \\ &\quad + (1-\lambda)[F(5) - F(-5)] \\ P(\text{"deflation"}) &= \lambda F(-.5) + (1-\lambda)F(-5) \end{aligned} \tag{3.19}$$

The log likelihood function is

$$\begin{aligned} \log L = {} & \Sigma_{S1}\log[\lambda F(-.5) + (1-\lambda)F(-5)] \\ & + \Sigma_{S2}\log[\lambda[F(.5) - F(-.5)] + (1-\lambda)[F(5) - F(-5)]] \\ & + \Sigma_{S3}\log(1-b) + \log[\lambda[1-F(.5)] + (1-\lambda)[1-F(5)]] \\ & + \Sigma_{S4A}\log b + \log\lambda + \log[F(\Pi^*+.5) - F(\Pi^*-.5)] \\ & + \Sigma_{S4B}\log b + \log[\lambda[F(\Pi^*+.5) - F(\Pi^*-.5)] \\ & \qquad + (1-\lambda)[F(\Pi^*+5) - F(\Pi^*-5)]] \end{aligned} \tag{3.20}$$

Employing the approximation in (3.5) gives the following approximate log likelihood

$$\begin{aligned} \log L \simeq {} & \Sigma_{S1}\log[\lambda F(-.5) + (1-\lambda)F(-5)] \\ & + \Sigma_{S2}\log[\lambda f(0) + (1-\lambda)[F(5) - F(-5)]] \\ & + \Sigma_{S3}\log(1-b) + \log[\lambda[1-F(.5)] + (1-\lambda)[1-F(5)]] \\ & + \Sigma_{S4A}\log b + \log\lambda + \log f(\Pi^*) \\ & + \Sigma_{S4B}\log b + \log[\lambda f(\Pi^*) + (1-\lambda)[F(\Pi^*+5) - F(\Pi^*-5)]] \end{aligned} \tag{3.21}$$

Table 3.1
Estimates of the Rounding Model

	Actual Inflation	μ	var μ	σ^2	λ
78:1	9.35	6.19	.1000	29.87	.7604
78:2	9.93	8.25	.1269	68.43	.7264
78:3	10.17	7.83	.2220	67.77	.7290
78:4	10.44	8.08	.2393	62.75	.7926
78:5	10.76	8.74	.2368	110.83	.6851
78:6	10.91	8.92	.3224	87.39	.7437
78:7	11.29	7.50	.1323	44.22	.7316
78:8	11.78	9.21	.1932	92.17	.6540
78:9	12.09	8.01	.1828	58.29	.7228
78:10	12.20	7.98	.1815	55.36	.7322
78:11	12.62	8.77	.1276	71.32	.6697
78:12	13.31	9.19	.3028	76.74	.7039
79:1	13.92	9.51	.2554	79.41	.7414
79:2	14.15	10.67	.2659	131.78	.6612
79:3	14.68	10.59	.3531	99.85	.7238
79:4	14.66	10.75	.2389	94.86	.6998
79:5	14.39	12.23	.2652	136.79	.6539
79:6	14.31	12.22	.3222	142.82	.6343
79:7	13.20	11.70	.2393	126.45	.6457
79:8	12.80	11.21	.2461	132.79	.6298
79:9	12.47	10.16	.2317	100.94	.6578
79:10	12.64	9.09	.1335	75.91	.6702
79:11	12.62	11.85	.2664	150.33	.6166
79:12	12.40	10.78	.3156	120.66	.6039
80:1	11.71	13.36	.6410	185.63	.6342
80:2	11.34	10.84	.1569	79.67	.6626

It can also be shown that the first derivatives of (3.21) are first-order Taylor's series expansions of the first derivatives of (3.20). Given the magnitude of the variances in Table 3.1 this approximation should be quite good. The first derivatives of (3.21) are

$$\partial \log L/\partial b = \Sigma_{S3} \frac{-1}{(1-b)} + \Sigma_{S4A} \frac{1}{b} + \Sigma_{S4B} \frac{1}{b} \tag{3.22}$$

$$\begin{aligned} \partial \log L/\partial \lambda \simeq\ & \Sigma_{S1} \frac{F(-.5)-F(-5)}{\lambda F(-.5)+(1-\lambda)F(-5)} \\ & + \Sigma_{S2} \frac{f(0)-[F(5)-F(-5)]}{\lambda f(0)+(1-\lambda)[F(5)-F(-5)]} \\ & + \Sigma_{S3} \frac{[1-F(.5)]-[1-F(5)]}{\lambda[1-F(.5)]+(1-\lambda)[1-F(5)]} \\ & + \Sigma_{S4A} \frac{1}{\lambda} \\ & + \Sigma_{S4B} \frac{f(\Pi^*)-[F(\Pi^*-5)-F(\Pi^*-5)]}{\lambda f(\Pi^*)+(1-\lambda)[F(\Pi^*+5)-F(\Pi^*-5)]} \end{aligned} \tag{3.23}$$

$$\begin{aligned} \partial \log L/\partial \mu \simeq\ & \Sigma_{S1} \frac{-\lambda f(-.5)-(1-\lambda)f(-5)}{\lambda F(-.5)+(1-\lambda)F(-5)} \\ & + \Sigma_{S2} \frac{\lambda f(0)(\frac{0-\mu}{\sigma^2})-(1-\lambda)[f(5)-f(-5)]}{\lambda f(0)+(1-\lambda)[F(5)-F(-5)]} \\ & + \Sigma_{S3} \frac{\lambda f(.5)+(1-\lambda)f(5)}{\lambda[1-F(.5)]+(1-\lambda)[1-F(5)]} \\ & + \Sigma_{S4A} \frac{(\Pi^*-\mu)}{\sigma^2} \\ & + \Sigma_{S4B} \frac{\lambda f(\Pi^*)(\frac{\Pi^*-\mu}{\sigma^2})-(1-\lambda)[f(\Pi^*+5)-f(\Pi^*-5)]}{\lambda f(\Pi^*)+(1-\lambda)[F(\Pi^*+5)-F(\Pi^*-5)]} \end{aligned} \tag{3.24}$$

$$
\begin{aligned}
\partial \log L/\partial\sigma^2 \simeq \Sigma_{S1} & \frac{-\frac{1}{2}\lambda(\frac{-.5-\mu}{\sigma^2})f(-.5)-\frac{1}{2}(1-\lambda)(\frac{-5-\mu}{\sigma^2})f(-5)}{\lambda F(-.5)+(1-\lambda)F(-5)} \\
+ \Sigma_{S2} & \frac{\lambda[-\frac{1}{2\sigma^2}+\frac{\mu^2}{2\sigma^4}]f(0)-\frac{1}{2}(1-\lambda)[(\frac{5-\mu}{\sigma^2})f(5)-(\frac{-5-\mu}{\sigma^2})f(-5)]}{\lambda f(0)+(1-\lambda)[F(5)-F(-5)]} \\
+ \Sigma_{S3} & \frac{\frac{1}{2}\lambda(\frac{.5-\mu}{\sigma^2})f(.5)+\frac{1}{2}(1-\lambda)(\frac{5-\mu}{\sigma^2})f(5)}{\lambda[1-F(.5)]+(1-\lambda)[1-F(5)]} \qquad (3.25) \\
+ \Sigma_{S4A} & -\frac{1}{2\sigma^2}+\frac{(\Pi^*-\mu)^2}{2\sigma^4} \\
+ \Sigma_{S4B} & \frac{\lambda[-\frac{1}{2\sigma^2}+\frac{(\Pi^*-\mu)^2}{2\sigma^4}]f(\Pi^*)-\frac{1}{2}(1-\lambda)[(\frac{\Pi^*+5-\mu}{\sigma^2})f(\Pi^*+5)-(\frac{\Pi^*-5-\mu}{\sigma^2})f(\Pi^*-5)]}{\lambda f(\Pi^*)+(1-\lambda)[F(\Pi^*+5)-F(\Pi^*-5)]}
\end{aligned}
$$

As before (3.22) can be solved for b as a sample proportion. The remaining equations were solved with the method of Berndt, Hall, Hall, and Hausman (1974). The estimates obtained are presented in Table 3.2. As expected, the estimates are quite similar to those in Table 3.1, with λ being somewhat larger.

When examining the impact of uncertainty on nominal interest rates, many studies have used the cross-section variance of expected inflation as a measure of the uncertainty with which individual expectations are held. See, for example, Bomberger and Frazer (1981), and Levi and Makin (1979). However, the cross-section variance measures differences in opinion across respondents, rather than the uncertainty with which an individual views his point estimate. It seems likely that a more-direct measure of uncertainty is provided by (1-λ), the probability that a randomly selected individual will provide a rounded response. The following example may help illustrate the point. If an individual wearing a watch is asked the time, something he will know with great accuracy, he may respond "9:51". If instead, someone without a watch is asked the time, something he knows with relative inaccuracy, he is likely to respond "about 10:00", even though he might respond "9:51" if pressed to provide a more-accurate response. That is, greater uncertainty about a point estimate is likely to result in more-severe rounding. If this is true, then surveys taken when respondents are more uncertain about expected inflation should produce higher estimation of $(1-\lambda)$.

It has been argued that situations in which individual forecasts differ greatly are likely to be situations in which individual forecasts are very uncertain. The values of λ and σ^2 reported in Table 3.2 provide a weak test of this proposition. If both $(1-\lambda)$ and σ^2 are measures of uncertainty, as one would suspect on intuitive grounds, then they should be positively correlated. The sample correlation coefficient between $(1-\lambda)$ and σ^2 is .8036. This suggests that in the absence of a direct measure of uncertainty, the cross section variance may be a useful proxy.

3.4 Estimation With Costless Predictors

The second alternative considers the impact of costless predictors on the distribution of responses. For the model with continuous predictions, and the most-recent price level as the costless predictor, substituting (3.6) and (3.7) into (3.12) gives

$$\begin{aligned} P(\Pi^*) &= b(1-q)f(\Pi^*) \\ P(\text{"inflation"}) &= (1-b)(1-q)[1-F(0)] \\ P(\text{"no inflation"}) &= q \qquad\qquad (3.26) \\ P(\text{"deflation"}) &= (1-q)F(0) \end{aligned}$$

Table 3.2
Estimates of the Integer Response Rounding Model

	μ	var μ	σ^2	λ
78:1	6.16	.0993	30.57	.8414
78:2	8.23	.1286	69.27	.8050
78:3	7.79	.2231	68.26	.8096
78:4	8.05	.2396	63.05	.8810
78:5	8.73	.2389	111.36	.7618
78:6	8.90	.3259	87.90	.8261
78:7	7.48	.1328	44.83	.8098
78:8	9.20	.1932	92.31	.7277
78:9	7.98	.1843	58.97	.8020
78:10	7.94	.1799	55.53	.8150
78:11	8.74	.1273	71.49	.7455
78:12	9.14	.3063	76.87	.7832
79:1	9.49	.2592	80.02	.8236
79:2	10.66	.2666	131.88	.7357
79:3	10.57	.3570	100.34	.8047
79:4	10.73	.2417	95.42	.7774
79:5	12.23	.2673	137.35	.7268
79:6	12.21	.3250	143.43	.7049
79:7	11.69	.2400	126.73	.7180
79:8	11.20	.2474	133.08	.7005
79:9	10.15	.2348	101.65	.7307
79:10	9.06	.1335	76.12	.7455
79:11	11.84	.2682	150.71	.6858
79:12	10.77	.3166	120.97	.6715
80:1	13.36	.6445	186.11	.7020
80:2	10.83	.1569	80.25	.7357

The log likelihood function is

$$\begin{aligned} \log L = {} & \Sigma_{S1} \log(1-q) + \log F(0) \\ & + \Sigma_{S2} \log q \\ & + \Sigma_{S3} \log(1-b) + \log(1-q) + \log[1-F(0)] \\ & + \Sigma_{S4} \log b + \log(1-q) + \log f(\Pi^*) \end{aligned} \tag{3.27}$$

The first derivatives are

$$\partial \log L/\partial b = \Sigma_{S3} \frac{-1}{(1-b)} + \Sigma_{S4} \frac{1}{b} \tag{3.28}$$

$$\partial \log L/\partial q = \Sigma_{S1} \frac{-1}{(1-q)} + \Sigma_{S2} \frac{1}{q} + \Sigma_{S3} \frac{-1}{(1-q)} + \Sigma_{S4} \frac{-1}{(1-q)} \tag{3.29}$$

$$\partial \log L/\partial \mu = \Sigma_{S1} \frac{-f(0)}{F(0)} + \Sigma_{S3} \frac{f(0)}{1-F(0)} + \Sigma_{S4} \frac{(\Pi^*-\mu)}{\sigma^2} \tag{3.30}$$

$$\begin{aligned} \partial \log L/\partial \sigma^2 = {} & \Sigma_{S1} \frac{f(0)}{F(0)} \frac{\mu}{2\sigma^2} \\ & + \Sigma_{S3} \frac{-f(0)}{1-F(0)} \frac{\mu}{2\sigma^2} \\ & + \Sigma_{S4} \frac{-1}{2\sigma^2} + \frac{(\Pi^*-\mu)^2}{2\sigma^4} \end{aligned} \tag{3.31}$$

The equations in (3.28) and (3.29) give, respectively, an estimate of b as the sample proportion of those responding "up" that were willing to give a point prediction, and an estimate of q as the sample proportion responding "no inflation". The remaining equations were solved for estimates of μ and σ^2 using the method of scoring. These equations are just a mixture of those in the familiar "Probit" and "Tobit" models. This occurs because point predictions are obtained from only part of those responding "up".

The estimates obtained are presented in Table 3.3, and are of the expected sign and magnitude. The estimates of μ are mean expected inflation for those using costly predictors. These estimates average about one percent higher than the estimates obtained with the rounding model, and are a better predictor of the actual inflation rate. Despite this improvement, these estimates also consistently underpredict the actual inflation rate.

In order to check the sensitivity of the estimates to the assumption that the costly predictors are continuous, estimates will be obtained for the integer response model. Substituting the probabilities of the integer response model in (3.8) and (3.9) into (3.12) gives the probability of the responses as

$$\begin{aligned} P(\Pi^*) &= b(1-q)[F(\Pi^*+.5) - F(\Pi^*-.5)] \\ P(\text{"inflation"}) &= (1-b)(1-q)[1-F(.5)] \\ P(\text{"no inflation"}) &= q + (1-q)[F(.5) - F(-.5)] \\ P(\text{"deflation"}) &= (1-q)\,F(-.5) \end{aligned} \tag{3.32}$$

The log likelihood function is

$$\begin{aligned} \log L = {} & \Sigma_{S1} \log(1-q) + \log F(-.5) \\ & + \Sigma_{S2} \log[q + (1-q)[F(.5) - F(-.5)]] \\ & + \Sigma_{S3} \log(1-b) + \log(1-q) + \log[1-F(.5)] \\ & + \Sigma_{S4} \log b + \log(1-q) + \log[F(\Pi^*+.5) - F(\Pi^*-.5)] \end{aligned} \tag{3.33}$$

Using the approximation in (3.5) gives the following approximate log likelihood.

$$\begin{aligned}\text{LogL} \simeq\ & \Sigma_{S1}\log(1-q) + \log F(-.5)\\ & + \Sigma_{S2}\log[q + (1-q)\ f(0)]\\ & + \Sigma_{S3}\log(1-b) + \log(1-q) + \log[1-F(.5)]\\ & + \Sigma_{S4}\log b + \log(1-q) + \log f(\Pi^*)\end{aligned} \tag{3.34}$$

It can be shown that the first derivatives of (3.34) are a first-order Taylor's series expansion of the first derivatives of (3.33). The first derivatives of (3.34) are:

$$\partial\log L/\partial b = \Sigma_{S3}\,\frac{-1}{(1-b)} + \Sigma_{S4}\,\frac{1}{b} \tag{3.35}$$

$$\begin{aligned}\partial\log L/\partial q \simeq\ & \Sigma_{S1}\,\frac{-1}{(1-q)}\\ & + \Sigma_{S2}\,\frac{[1-f(0)]}{[q+(1-q)f(0)]}\\ & + \Sigma_{S3}\,\frac{-1}{(1-q)}\\ & + \Sigma_{S4}\,\frac{-1}{(1-q)}\end{aligned} \tag{3.36}$$

$$\begin{aligned}\partial\log L/\partial \mu \simeq\ & \Sigma_{S1}\,\frac{-f(-.5)}{F(-.5)}\\ & + \Sigma_{S2}\,\frac{(1-q)(\frac{0-\mu}{\sigma^2})f(0)}{[q + (1-q)\ f(0)]}\\ & + \Sigma_{S3}\,\frac{f(.5)}{1-F(.5)}\\ & + \Sigma_{S4}\,\frac{(\Pi^*-\mu)}{\sigma^2}\end{aligned} \tag{3.37}$$

$$\partial\log L/\partial\sigma^2 \simeq \Sigma_{S1} \frac{-f(-.5)}{F(-.5)} \frac{(-.5-\mu)}{2\sigma^2}$$

$$+ \Sigma_{S2} \frac{(1-q)f(0)[-\frac{1}{2\sigma^2} + \frac{(0-\mu)^2}{2\sigma^4}]}{[q + (1-q)f(0)]}$$

$$+ \Sigma_{S3} \frac{f(.5)}{1-F(.5)} \frac{(.5-\mu)}{2\sigma^2} \tag{3.38}$$

$$+ \Sigma_{S4} \frac{-1}{2\sigma^2} + \frac{(\Pi^*-\mu)^2}{2\sigma^4}$$

Equation (3.35) gives an estimate of b as a sample proportion. Equation (3.36) can be solved for

$$\frac{N_1+N_3+N_4}{N_2[1-f(0)]} = \frac{(1-q)}{q+(1-q)f(0)} \tag{3.39}$$

This result can be used to eliminate q from the remaining equations. Multiplying both sides of (3.39) by f(0) gives an expression for the probability that an individual predicting no inflation has invested in a costly predictor. Equations (3.37) and (3.38) are analogous to those of the preceding model, except the observations at zero inflation appear, and are weighted by the probability they were generated by a costly predictor. These equations were solved for estimates of μ and σ^2 using the method of scoring. The estimates obtained are presented in Table 3.4. As expected, these estimates are quite similar to those in Table 3.3, with q somewhat smaller.

To this point, the most-recent price level was used as a costless predictor of the current price level. An alternative costless predictor is the use of the most-recent inflation rate to predict the current inflation rate, and to construct a price level predictor accordingly. There is now some probability p, of randomly selecting an individual that is using the most-recent inflation rate as a costless predictor,

Table 3.3
Estimates of Model With Most-Recent Price Level
as the Costless Predictor

	Actual Inflation	μ	var μ	σ^2	q
78:1	9.35	6.92	.063	32.98	.099
78:2	9.93	9.21	.074	73.82	.097
78:3	10.17	8.73	.119	72.77	.098
78:4	10.44	8.57	.110	64.47	.055
78:5	10.76	9.79	.110	117.41	.104
78:6	10.91	9.67	.161	91.30	.073
78:7	11.29	8.25	.080	48.31	.082
78:8	11.78	10.22	.097	94.54	.097
78:9	12.09	8.97	.107	62.07	.102
78:10	12.20	8.73	.095	56.35	.087
78:11	12.62	9.68	.063	72.99	.094
78:12	13.31	9.66	.134	79.82	.049
79:1	13.92	10.04	.111	81.33	.050
79:2	14.15	11.60	.115	134.29	.079
79:3	14.68	11.03	.155	101.89	.039
79:4	14.66	11.36	.121	97.45	.053
79:5	14.39	13.09	.128	139.10	.063
79:6	14.31	13.35	.165	145.50	.083
79:7	13.20	12.85	.129	126.97	.087
79:8	12.80	12.54	.137	135.16	.104
79:9	12.47	11.25	.136	104.58	.093
79:10	12.64	10.14	.084	77.18	.102
79:11	12.62	13.25	.141	152.95	.102
79:12	12.40	12.23	.176	122.47	.116
80:1	11.71	14.49	.288	189.01	.076
80:2	11.34	11.72	.092	80.75	.075

Table 3.4
Estimates of Integer Response Model With
Most-Recent Price Level as Costless Predictor

	μ	var μ	σ^2	q
78:1	6.78	.0628	33.19	.0818
78:2	9.08	.0733	73.84	.0838
78:3	8.59	.1194	72.62	.0846
78:4	8.41	.1118	64.41	.0404
78:5	9.67	.1097	116.98	.0921
78:6	9.52	.1599	91.16	.0611
78:7	8.13	.0801	48.54	.0685
78:8	10.11	.0971	94.57	.0858
78:9	8.83	.1083	62.21	.0899
78:10	8.61	.0966	56.54	.0739
78:11	9.55	.0637	73.13	.0827
78:12	9.51	.1336	79.77	.0370
79:1	9.92	.1122	81.95	.0390
79:2	11.49	.1142	134.20	.0699
79:3	10.90	.1555	101.96	.0291
79:4	11.25	.1230	97.67	.0430
79:5	12.98	.1287	139.31	.0551
79:6	13.25	.1663	145.71	.0749
79:7	12.75	.1294	127.26	.0796
79:8	12.44	.1371	135.30	.0957
79:9	11.15	.1363	104.73	.0840
79:10	10.03	.0846	77.39	.0936
79:11	13.14	.1410	153.06	.0948
79:12	12.14	.1760	122.66	.1084
80:1	14.41	.2887	189.09	.0693
80:2	11.63	.0930	81.18	.0676

Table 3.5
Estimates of Model With Both Most-Recent Price Level and Most-Recent Inflation Rate as Costless Predictors

	μ	var μ	σ^2	p
78:1	6.96	.077	36.78	.077
78:2	9.39	.081	77.23	.038
78:3	9.04	.152	82.47	.091
78:4	8.98	.153	76.12	.119
78:5	10.21	.140	132.26	.094
78:6	10.05	.199	101.57	.083
78:7	8.28	.089	51.09	.042
78:8	10.33	.105	98.52	.034
78:9	9.04	.118	65.06	.035
78:10	8.75	.108	60.35	.052
78:11	9.75	.071	77.66	.047
78:12	9.74	.148	84.24	.039
79:1	10.16	.129	88.31	.061
79:2	12.10	.182	169.17	.175
79:3	11.43	.227	123.88	.151
79:4	11.80	.182	119.72	.156
79:5	13.16	.137	143.57	.028
79:6	13.45	.176	150.58	.029
79:7	12.91	.137	130.72	.024
79:8	12.56	.144	138.50	.019
79:9	11.24	.149	109.54	.036
79:10	10.08	.091	80.41	.033
79:11	13.30	.168	167.03	.067
79:12	12.19	.203	131.73	.057
80:1	14.50	.291	190.20	.005
80:2	11.69	.094	81.61	.010

*The q series is just that presented in Table 1.

and will always respond "Π_{t-1} inflation". There is still the probability q of randomly selecting an individual who is using the most-recent price level as a costless predictor and will always respond "no inflation."

The log likelihood function for the model with continuous predictions is now

$$\begin{aligned} \log L = {} & \Sigma_{S1} \log(1-q-p) + \log F(0) \\ & + \Sigma_{S2} \log q \\ & + \Sigma_{S3} \log(1-b) + \log(1-q-p) + \log[1-F(0)] \\ & + \Sigma_{S4C} \log b + \log(1-q-p) + \log f(\Pi^*) \\ & + \Sigma_{S4D} \log p \end{aligned} \tag{3.40}$$

where S4C is the set of elements in S4 not equal to Π_{t-1}, and S4D is the set of elements in S4 equal to Π_{t-1}. The first derivatives give estimates of b, q, and p as sample proportions. Estimates of μ and σ^2 are obtained by applying equations (3.30) and (3.31) to the observations in S1, S3, and S4C. The estimates obtained with this model are presented in Table 3.5, and are quite similar to those in Table 3.3.

3.5 Summary

Maximum-likelihood estimates that account for the censoring that occurs in both the Deflation and Inflation categories, have been presented for each of the alternative explanations of the discontinuity in the observed distribution of responses. The first alternative is based on the presence of rounded responses. The estimates obtained from this model can also be used to examine the performance of the cross-section variance as a measure of uncertainty. These estimates suggest that in the absence of a direct measure of uncertainty, the cross-section variance may provide a useful proxy. The second alternative is based on the presence of costless predictors of expected prices. The estimates of this model provide a measure of expected inflation for a relatively informed group, those using costly predictors. The estimates of expected inflation for this model average about one percent higher than the estimates obtained from the model with rounded responses, and are a better predictor of the actual inflation rate. However, both series consistently underpredict the actual inflation rate. The root mean square error with the actual inflation rate is 2.27 for the model with costless predictors, and 2.91 for the model with rounded responses. However, the fact that the series for the costless estimator model is a better predictor of the actual inflation rate does not mean it is a more-accurate measure of expected inflation.

A discussion of the relative merits of these alternative estimates must be delayed until the series are applied in Chapter IV. One of the incentives for attempting to distinguish between the behavior of costly and costless estimators was the observation that market variables might reflect the expectations of an informed group rather than the expectations of the general population. Thus, application of these estimates should help us evaluate the different series.

CHAPTER IV
ESTIMATION OF A MACROECONOMIC MODEL USING THE SRC EXPECTED INFLATION SERIES

4.1 The Macroeconomic Model

The expected inflation series obtained in the previous chapter will be used to estimate the following macro model.

$$Y_t = \overline{Y}_t + \alpha(\Pi_t - \Pi_t^*) + u_{1t} \tag{4.1}$$

$$M_t = P_t + Y_t + \beta r_t + u_{2t} \tag{4.2}$$

$$Y_t = \overline{Y}_t + \gamma[r_t(1-\tau) - \Pi_t^*] + \delta A_t + u_{3t} \tag{4.3}$$

where Y_t is the log of real output, $\overline{Y}_t$ is the log of "natural" real output, Π_t is the actual inflation rate, Π_t^* is the expected inflation rate, M_t is the log of the nominal money stock, P_t is the log of the price level, r_t is the nominal interest rate, and A_t represents other variables influencing expenditure. The u_{it} are disturbances.

Equation (4.1) is an expectations adjusted Phillip's curve. Equation (4.2) is a portfolio balance equation. Equation (4.3) is an expenditure function, where real expenditure is a function of the real rate of interest. Statistically, the term $(1-\tau)$ in equation (4.3) allows the coefficients of the nominal interest rate and expected inflation to differ. A theoretical motivation for this term is given in Darby (1975). τ is the marginal tax rate on interest income, and (4.3) states that real expenditure depends on the real after-tax rate of interest. The system given in (4.1) through (4.3) is similar to those in Sargent (1973) and Sargent (1979).

The endogenous variables are real output, the price level, and the nominal interest rate. The exogenous variables are expected inflation, the nominal money stock, the lagged values of the endogenous variables. Theory suggests that $\alpha > 0$, $\beta < 0$, and $\gamma < 0$.

There are two questions of particular interest with this system. First, what is the impact of a change in expected inflation on nominal interest rates? And second, do only unanticipated changes in the price level influence output? These questions illustrate the importance of the expected inflation series.

Because we are dealing with a simultaneous system, the effect of expected inflation on nominal interest rates is determined by the coefficient of expected inflation in the reduced form equation for nominal interest rates. Recalling that P_t is log of the price level, substitute the equation $\Pi_t = P_t - P_{t-1}$ into (4.1) and solve the system for the reduced form equation.

$$r_t = \frac{\gamma(1+\alpha)-\alpha}{\beta\alpha+\gamma(1-\tau)(1+\alpha)}\,\Pi_t^* + \frac{\alpha(M_t-P_{t-1}-\overline{Y}_t)}{\beta\alpha+\gamma(1-\tau)(1+\alpha)}$$

$$- \frac{\delta(1+\alpha)A_t}{\beta\alpha+\gamma(1-\tau)(1+\alpha)} + \frac{u_{1t}-\alpha u_{2t}-(1+\alpha)u_{3t}}{\beta\alpha+\gamma(1-\tau)(1+\alpha)} \qquad (4.4)$$

Thus, we have an equation of the form

$$r_t = C\Pi_t^* + DA_t + E(M_t - P_{t-1} - \overline{Y}_t) + \varepsilon_t \qquad (4.5)$$

Given the expected signs of the structural coefficients, we have $C > 0$ and $E < 0$.

The result just presented is short run in the sense that it allows expected and actual inflation to differ, and thus allows output to change. The long-run effect, where inflation is as expected and output returns to the "natural" level, can be obtained by restricting $\alpha = 0$. With this restriction, the coefficient C is reduced to $\frac{1}{(1-\tau)}$. Because the data exhibits variation in output, we cannot estimate the long-run effect directly from the reduced form equation. We need an equation for nominal interest rates where fluctuations in output are accounted for. Consider the expenditure equation in this macroeconomic model normalized on r_t rather than Y_t.

$$r_t = \frac{1}{(1-\tau)} \Pi_t^* + \frac{1}{\gamma(1-\tau)} (Y_t - \overline{Y}_t) - \frac{\delta}{\gamma(1-\tau)} A_t - \frac{1}{\gamma(1-\tau)} u_{3t} \qquad (4.6)$$

Note that the coefficient of expected inflation is the same as that in the reduced form equation for nominal interest rates under the restriction $\alpha = 0$. This is a very popular equation and has been estimated by Tanzi (1980) and by Levi and Makin (1979) using single-equation methods. However, this is a structural equation and cannot be consistently estimated with single-equation methods. Finally, note that this equation is identified.

To examine whether only unexpected inflation influences output, the following equation will be estimated.

$$\Pi_t = \Pi_t^* + \frac{1}{\alpha}(Y_t - \overline{Y}_t) - \frac{1}{\alpha} u_{1t} \qquad (4.7)$$

This is just the expectations adjusted Phillip's curve normalized on Π_t instead of Y_t. If only unexpected inflation influences output, then the coefficient of expected inflation, should be one. This equation is also identified.

4.2 Estimation of the Macroeconomic Model

Equation (4.5) will be estimated to examine the initial impact of expected inflation on nominal interest rates. Equation (4.6) will be estimated with two-stage least-squares to examine the long-run impact of expected inflation on nominal interest rates. Equation (4.7) will be estimated with two-stage least-squares to test the proposition that only unexpected inflation influences output. Both the rounding series and the costless estimator series will be used to measure expected inflation. The CPI (1967 = 100) was used for the price level, M1A was used for the nominal money stock, and one year Treasury Bill rate was used for the nominal interest rate, the Federal Reserve Board's series for industrial production and capacity industrial production was used for output and "natural" output, and A_t was initially ignored.

The estimates obtained are presented in Table 4.1. Given that we are modeling an aggregate economy on a monthly basis, and given the relatively simple specification of the model being used, the estimates should be interpreted with caution. The signs of these estimates are as expected. The estimates of the reduced form interest rate equation give the initial impact of a one percent increase in expected inflation as, a .16 percent increase in nominal interest rates with the rounding series, and as a .15 percent increase with the costless predictor series. Notice however, that neither of these coefficents are significantly different from zero. This insignificance probably occurs because the economy is being modeled on a monthly basis. The estimates above give the response of nominal interest rates to a change in expected inflation during the month of the change. It is quite possible that it takes the economy longer to respond. The long-run importance of expected inflation

Table 4.1
Estimates of the Macroeconomic Model

	Variable	Coefficient	St. Dev.	t ratio
Reduced Form Interest Equation				
Rounding Series	intercept	-.3898	.0836	-4.66
	π^*_t	.1575	.1945	.81
	$(M_t-P_{t-1}-\overline{Y}_t)$	-.3809	.0795	-4.79
Costless Predictor Series	intercept	-.3890	.0850	-4.58
	π^*_t	.1453	.1834	.79
	$(M_t-P_{t-1}-\overline{Y}_t)$	-.3802	.0812	-4.68
Expenditure Equation				
Rounding Series	intercept	-.1618	.1066	-1.52
	π^*_t	1.1221	.2430	4.62
	$(Y_t-\overline{Y}_t)$	-.8651	.5431	-1.59
Costless Predictor Series	intercept	-.0825	.1107	- .75
	π^*_t	.9052	.2154	4.20
	$(Y_t-\overline{Y}_t)$	-.4592	.5796	- .97
Expectations Adjusted Phillip's Curve				
Rounding Series	π^*_t	1.2968	.2243	5.78
	$(Y_t-\overline{Y}_t)$	.0994	.1379	.72
Costless Predictor Series	π^*_t	1.1869	.2178	5.45
	$(Y_t-\overline{Y}_t)$	.0997	.1463	.68

in determining nominal interest rates is verified by the estimates of the expenditure equation. These estimates give the long-run impact of a one percent increase in expected inflation as, a 1.12 percent increase in nominal interest rates with the rounding series, and as a .91 percent increase with the costless predictor series. Both these coefficients are significantly different than zero, but not significantly different than one. Finally, with either expected inflation series, the coefficient of expected inflation in the expectations adjusted Phillip's curve is not significantly different than one. This tends to support the idea that only unexpected inflation influences output. Notice however, that the coefficient of the difference between output and capacity output, while of the expected sign, is not significantly different than zero. Once again, this may reflect the fact that a month is a very short time for aggregate supply to respond.

Lucas (1976) has shown that changes in uncertainty should result in changes in the structural coefficients. These functions are generally non-linear, and recent studies such as Bomberger and Frazer (1981) and Levi and Makin (1979) have simply added an uncertainty measure linearly as an explanatory variable. In the model presented here, this can be accomplished by letting A_t in the expenditure equation represent the cross-section variance of expected inflation. The results obtained are presented in Table 4.2. As before, the estimates are of the expected sign.

The presence of the uncertainty measure drastically increases the coefficient of expected inflation in both the reduced form interest rate equation and the expenditure equation. In fact, the coefficient of expected inflation in the expenditure equation is so large that the

Table 4.2
Estimates of the Macroeconomic Model With Uncertainty

Reduced Form Interest Equation

	Variable	Coefficient	St. Dev.	t ratio
Rounding Series	intercept	- .3723	.0830	-4.49
	Π_t^*	.6355	.3999	1.59
	$(M_t-P_{t-1}-\overline{Y}_t)$	- .3446	.0825	-4.18
	σ^2	-2.0904	1.5369	-1.36
Costless Predictor Series	intercept	- .3407	.0847	-4.02
	Π_t^*	.8443	.4144	2.04
	$(M_t-P_{t-1}-\overline{Y}_t)$	- .3043	.0872	-3.49
	σ^2	-3.1086	1.6728	-1.86

Expenditure Equation

Rounding Series	intercept	- .2771	.0946	-2.93
	Π_t^*	2.6176	.5128	5.10
	$(Y_t-\overline{Y}_t)$	-1.0994	.4171	-2.64
	σ^2	-7.2241	2.0623	-3.50
Costless Predictor Series	intercept	- .1474	.0851	-1.73
	Π_t^*	2.1483	.4325	4.96
	$(Y_t-\overline{Y}_t)$	- .4533	.3919	-1.16
	σ^2	-6.003	1.9816	-3.43

Expectations Adjusted Phillip's Curve

Rounding Series	Π_t^*	1.2712	.5205	2.44
	$(Y_t-\overline{Y}_t)$	.0898	.1821	.49
	σ^2	.1021	2.7704	.04
Costless Predictor Series	Π_t^*	1.1218	.5374	2.09
	$(Y_t-\overline{Y}_t)$	.0730	.2017	.36
	σ^2	.2703	3.0356	.09

estimates must be viewed with some skepticism. Especially in view of the ad hoc nature in which uncertainty has been introduced into the estimation process. Finally, when introduced into the Phillip's curve, the uncertainty measure was insignificant and had little impact on the other coefficients.

4.3 Summary and Conclusions

The expected inflation series obtained in the previous chapter have been used to estimate a three equation macro model. This model contained an expectations adjusted Phillip's curve, a portfolio balance equation, and an expenditure equation. This system is used to examine both the initial and long-run impact of expected inflation on nominal interest rates. The long-run impact can be measured by one of the structural parameters, and must be estimated with systems methods. Finally, the proposition that only unexpected inflation influence output can be tested.

The estimates obtained suggest that the initial increase in nominal interest rates following a one percent increase in expected inflation is about .15 percent. While the long-run increase is not significantly different than one. The estimates also support the existence of the expectations adjusted Phillip's curve.

There is little difference between the estimates obtained with the alternative expected inflation series. This suggests one of two alternatives. Either, the economic variables considered reflect the expectations of the general population, or the informed group whose expectations are measured with the costless predictor series is not the relevant group for modeling purposes. Which of these alternatives is true, is unimportant. They suggest that there is little reason to prefer the expectations of this informed group over those of the general population, despite the fact that the expectation of the informed group were more accurate. This observation, and the fact that the model with rounded responses can explain the discontinuities at non-zero integer multiples of ten as well, suggests that it may be the preferred model.

BIBLIOGRAPHY

Berndt, E. K., B. H. Hall, R. E. Hall, and J. A. Hausman (1974): "Estimation and Inference in Non-Linear Structural Models," Annals of Economic and Social Measurement, 3, 653-665.

Bomberger, W. A., and W. J. Frazer (1981): "Interest Rates, Uncertainty, and the Livingston Data," Journal of Finance, 36, 661-675.

Carlson, J. A. (1977): "A Study of Price Forecasts," Annals of Economic and Social Measurement, 1, 27-52.

Darby, M. R. (1975): "The Financial and Tax Effects of Monetary Policy on Interest Rates," Economic Inquiry, 13, 266-276.

deMenil, G., and S. Bhalla (1975): "Direct Measurement of Popular Price Expectations," American Economic Review, 65, 169-180.

Diewert, W. E. (1974): "Applications of Duality Theory," Frontiers of Quantitative Economics II, ed. by M. Intrilligator and D. Kendrick, North Holland, Amsterdam.

Fackler, J., and B. Stanhouse (1977): "Rationality of the Michigan Price Expectation Data," Journal of Money, Credit, and Banking, 9, 662-666.

Fishe, R. P. H., and K. Lahiri (1981): "On the Estimation of Inflationary Expectations from Qualitative Responses," Journal of Econometrics, 16, 89-102.

Juster, T. (1973): "Savings Behavior, Uncertainty and Price Expectations," Survey of Consumers: 1972-1973, ed. by B. Strumpel, Institute for Social Research, University of Michigan, Ann Arbor.

Juster, T., and R. Comment (1978): "A Note on the Measurement of Price Expectations," unpublished report, University of Michigan.

Levi, M. D., and J. H. Makin (1979): "Fisher, Phillips, Friedman and the Measured Impact of Inflation on Interest," Journal of Finance, 34, 35-52.

Lindgren, B. W. (1976): Statistical Theory, Macmillan, New York.

Lucas, R. E. (1976): "Econometric Policy Evaluation: A Critique," The Phillips Curve and Labor Markets, Carnegie-Rochester Conference Series on Public Policy, 1, 19-46.

Maddala, G. S., R. P. H. Fishe, and K. Lahiri (1981): "A Time Series Analysis of Popular Expectations Data," Discussion Paper No. 62, University of Florida.

Mullineaux, D. J. (1978): "On Testing for Rationality: Another Look at the Livingston Price Expectations Data," Journal of Political Economy, 86, 329-336.

Pearce, D. K. (1979): "Comparing Survey and Rational Measures of Expected Inflation," Journal of Money, Credit, and Banking, 11, 447-456.

Sargent, T. J. (1973): "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment," Brookings Papers on Economic Activity, 2, 429-472.

Sargent, T. J. (1979): Macroeconomic Theory, Academic Press, New York.

Tanzi, V. (1980): "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates," American Economic Review, 70, 12-22.

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Thomas Zuehlke was born in Springfield, Illinois, in 1957. He received a Bachelor of Arts degree in economics from the University of South Florida in 1978. A Master of Arts degree was obtained at the University of Florida in 1979, and this document represents partial requirements for a Doctor of Philosophy degree expected in December 1983.

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